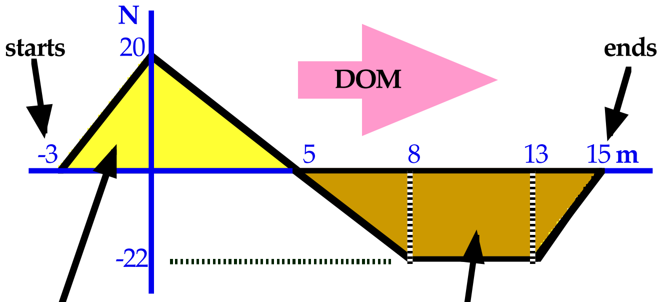
HOW THE UNIVERSE WORKS -- SIDE 34

example #38a: Work problem using area under a "curve" for an object moving left to right:

How much work is represented by the graph below?



Even though the object starts to the left of the origin, it still represents pos. work since Δx is pos.

This trapezoid represents negative work by the varying force on the object.

$W = 1/2(b_1 + b_2)\ h$

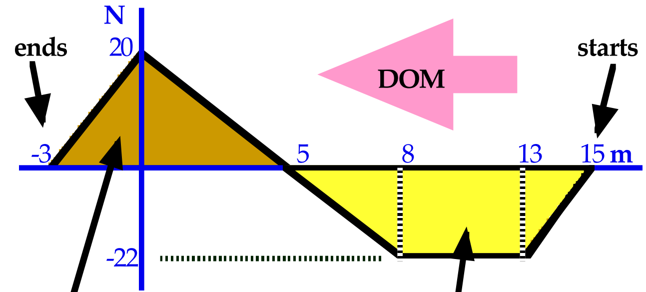
W = 1/2 b h = (.5)(8m)(20N) = +80 N m

W = 1/2(10m + 5m)(-22N)
W = -165 N m

ΣW = -85 Nm **so this varying force(s) drained 85 Joules of energy from the system.**

example #38b: Work problem using area under a "curve" for an object moving right to left:

How much work is represented by the graph below?



By the same logic, this area is neg.

This seems weird, but now this area is positive. because you are adding up the area from right to left. Up until now, we have always added the area from left to right because we were dealing with time which always increases left to right on a # line.

If we follow example 32a, we get a positive 165Nm worth of work poured into the system by the *negative* varying force(s) moving the object backwards from the 15 m mark to the 5 m mark (since my Δx is now -10m). I realize this is strange and a little counterintuitive, but sometimes you have to blindly trust in the math since it is mathematical relationships that are the only real physical truths in life. It's what separates science from religion. Anyway . . . the negative force(s) have poured in 165 Nm of positive work into this system from 15m to 5 m then something happens at the 5 m mark -- notice we have no idea when it happened since time is not a factor in the work graphs-- the force(s) switch over to the positive side and grow to a maximum positive influence until it gets to the origin, then it fades away as it approaches the -3m mark. From 5 m to -3m, the varying force(s) drains 80 Joules of energy from the system. Overall; however, energy is gained {165 J - 80 J = +85 J } during this object's journey from 15m to (-3m).

The Rate of Work (or . . . how quickly energy is draining out or filling up)

Average Power: (the rate of work) $\overline{P} = \dfrac{W}{\Delta t}$ *or* $\overline{P} = F \bullet \overline{v}$ $(dotproduct)$ or . . .

Alternative equation: $\overline{P} = |F|\ |\overline{v}|\ \cos\theta$ Where θ is the angle between the Force & the avg. vel. vector.

Instantaneous Power: $P = \mathbf{F} \cdot \mathbf{v}$ (dot product) or $P = |\mathbf{F}|\ |\mathbf{v}|\ \cos\ \theta$

The metric unit of power is Watts (W) English unit is horsepower (hp)

$1\ W = 1\ N \cdot m\ /\ s$ $\quad 1\ hp = 550\ ft \cdot lbs\ /\ s$ $\quad$ Conversion: 746 Watts = 1 hp

example #39a Instantaneous power example:

A car is driven at a steady rate of **34 m/s** down a flat road. The car's engine delivers **270 horsepower**. What is the overall frictional force resisting the motion of the car?

Looks like I will need a quick convertion to start:

$\dfrac{270\ hp}{} \Big| \dfrac{746\ W}{hp} = 201{,}420$ Watts = Power

This is a force equilibrium problem as well as a power problem.

$\Sigma F_x = 0$ since there is no acceleration $\therefore\ f = -F_p$

Also, since the speed is steady, $\overline{v} = v$ and the angle between the force moving the car forward and the velocity vector is 0°, I can simply write that $P = F_p\ v \quad \therefore \quad F_p = P\ /\ v$

$\therefore\ f = -(P\ /\ v) = -(201{,}420\ W \div 34\ m/s) =$ **5,924 N**

example #39b Average power example:

Preston (**1200 N**) runs up the North Gym steps at a steady rate wearing a **700 N** backpack full of weights. If each step is **20cm** tall, determine his <u>horsepower</u> as he runs **14 steps** in **3.1 secs.**

This activity should sound familiar. Here we are looking for his average hp. I'll figure it out in Watts then switch to hp. I am going to make the assumption that the amount of work required for Preston to move his body horizontally is small compared to the work required to fight gravity and move him to the next higher step. So $\overline{P} = (F\ \Delta y)\ /\ \Delta t$ =

= (1200 + 700)N (14 steps · 0.20 m/step) / 3.1 sec = 1716 W

$\dfrac{1716\ W}{} \Big| \dfrac{1\ hp}{746\ W}$ = **2.3 hp**

HOW THE UNIVERSE WORKS -- SIDE 35

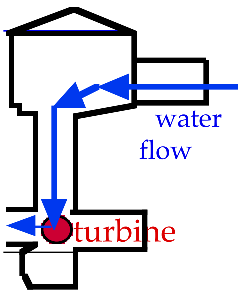
We have recently uncovered a new special case equation for Power:

After some manipulation of the power equation and what we know of derivatives, we can develop a new equation for power resulting from harnessing falling water: Starting with the equation for instantaneous power.

$$P = F \bullet v = Fv\cos\theta \; assume \; F \; and \; v \; are \; in \; the \; same \; downward \; direction \; then$$

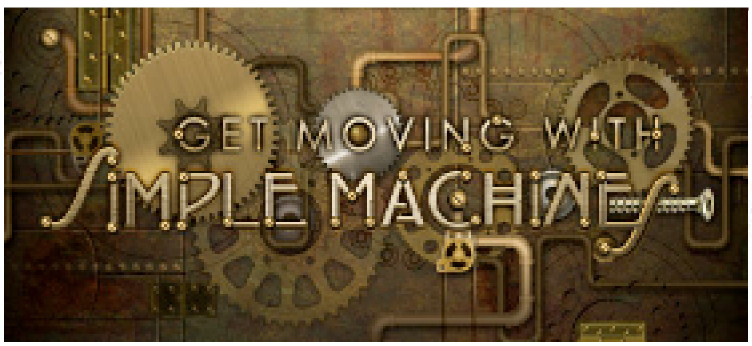
$$P = Fv = mg(dh/dt) \; \text{since} \; the \; water \; is \; falling \; from \; a \; height = (dm/dt)gh_i = \dot{m}gh_i$$

This "m dot" should look familiar to you from the Thrust Equation we developed back in the rocketry section on side 16. Remember that this flow rate is the amount of mass of a gas or fluid going past a certain point per unit of time.



example #40 Power of Niagara Falls: In 1890, George Westinghouse, used some "back of the envelope" calculations as he looked over Niagara Falls to determine approximately how much power he could produce using some diverted falling water and the latest in AC technology, Nikola Tesla's polyphase induction motor. He determined a diverted water flow rate of **1.2Gg/sec** and a vertical fall of **50 m** down the shaft shown to the left. Use your new equation to determine how many MW of power that generator would produce.

$$P = \left(\frac{dm}{dt}\right)gh = (1.2Gg/s)(9.81m/s/s)(50m) = \boxed{589MW}$$



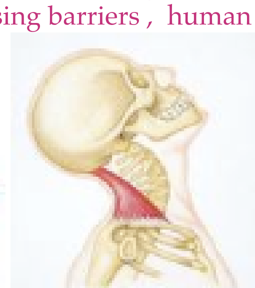
You might have spent some time in 9th grade working with simple machines, but there's a lot more to it than that. It is probably the most practical topic we will study this year. Machines give you some type of advantage to move an object through a distance. Simple machines will either provide a force advantage or a distance advantage, but one simple machine can not do both. Due to the 1st Law of Thermodynamics (Conservation of Energy) there is always a price to pay for any advantage

The "load" or "resistance" that the machine is trying to do work on is the "Force out". The "effort" you or another machine is putting into a machine is the "Force in".

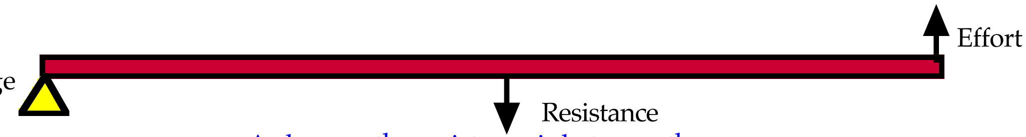
1st class levers:

As long as the fulcrum is between the effort and the resistance, it is a 1st class lever.

Examples: teeter totters, crow bars, hammers (when pulling out a nail), double pan balances, pulleys, skate boards (when braking), scissors, plyers, rail road crossing barriers , human arm (when bending at the elbow), etc.



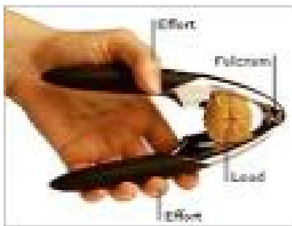
2nd class levers:

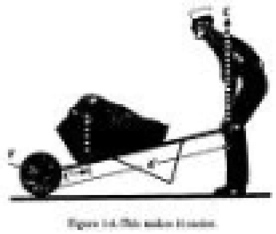


Gives you a force advantage
The price you pay:
distance disadvantage

As long as the resistance is between the effort and the fulcrum, it is a 2nd class lever.

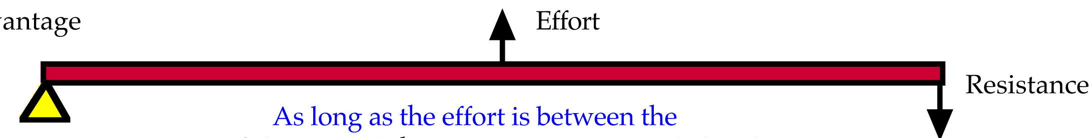
Examples: nutcracker, wheelbarrow, wrench, bottle opener, spring diving board, oars, foot when lifting up on toes





3rd class levers:

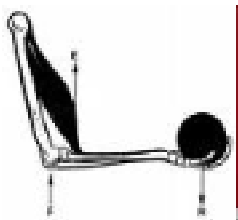
Gives you a distance advantage



The price you pay:
force disadvantage

As long as the effort is between the fulcrum and the Resistance, it is a 3rd class lever.

Examples: Most sports "sticks" (baseball bats, golf clubs, tennis racquet, etc.), arm holding a weight, stapler, tongs



A good pneumonic to use to remember which of the components of each lever is in the middle is "Free 123" It makes more sense if it's written $\frac{F\,R\,E}{1\,2\,3}$. This stands for fulcrum in the middle for a 1st class lever, resistance in the middle for a 2nd class lever, and effort in the middle for a 3rd class lever.

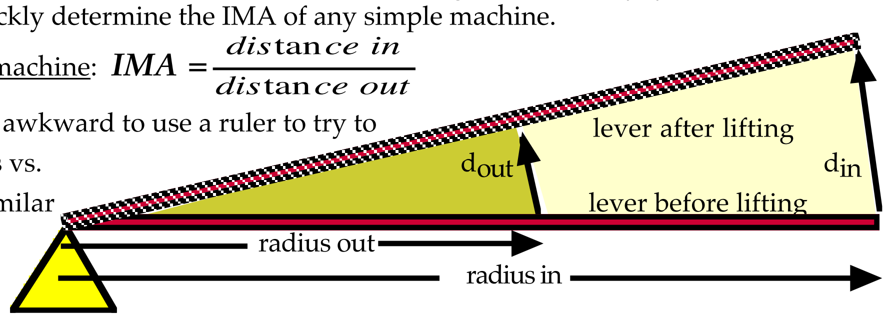
For an ideal, perfect, frictionless machine ***Work in = Work out***. Expanding this: $F_{in}d_{in} = F_{out}d_{out}$

Rearranging this *for an ideal machine* we have the opening equation for mechanical advantage: $\frac{d_{in}}{d_{out}} = \frac{F_{out}}{F_{in}}$

Now, the whole point to a simple machine is to give you some advantage. Mechanical advantage is a rating system that seems to be geared towards a force advantage. In fact, it seems like a distance advantage (3rd class levers, and some 1st), which can be equally useful, doesn't get much respect. Distance comparison leads to Ideal Mechanical Advantage (IMA) which is easier to determine than Actual Mechanical Advantage. It is so easy, you can use a ruler and a little similar triangle geometry to quickly determine the IMA of any simple machine.

Ideal Mechanical Advantage of a simple machine: $IMA = \frac{distance\ in}{distance\ out}$

So IMA involves distances, but it would be awkward to use a ruler to try to measure the distance the input force moves vs. the distance the resistive force moves. Similar triangles on this 2nd class lever will show us a much simpler way.

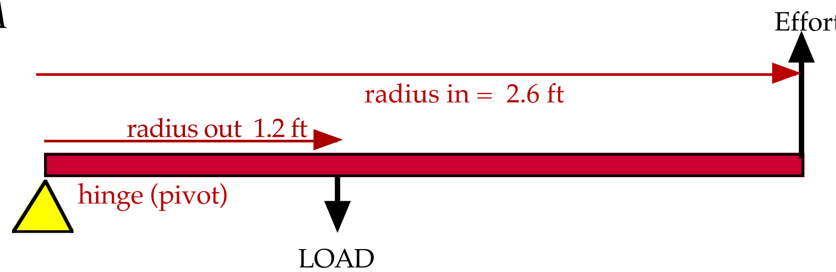


Refer to the figure at the bottom of the previous page of a 2nd class lever being rotated about its pivot (fulcrum). The effort force at the right end lifts through a distance we call d_{in} and the load or resistance force lifts through a smaller distance d_{out}. Notice a smaller darker triangle forms which is imbedded in a larger triangle. By the AAA (angle-angle-angle) theorem from geometry we have two similar triangles sharing a common angle at the fulcrum. Therefore we can compare the triangle's corresponding sides and come up with this very helpful relationship:

$$\frac{distance\ in}{distance\ out} = \frac{radius\ in}{radius\ out} = IMA$$

example #41 IMA of a 2nd class lever

Determine the Ideal Mechanical Advantage of a steel rod represented by the figure to the right.

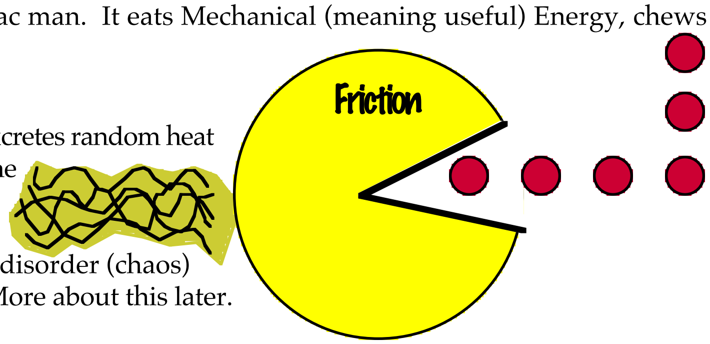


$$IMA = \frac{radius\ in}{radius\ out} = \frac{2.6\,ft}{1.2\,ft} = 2.16\ (no\ units)$$

So, the IMA is 2.16 So what? This simple ruler measurement tells me that I can lift 2.16 times the amount of lbs of force I put in at the end of the 2.6 foot pole. So with 100 lbs of force I can lift 216 lbs. Not bad eh? Unfortunately, remember, there is always a price to pay: to lift the load, say 4 feet, I have to pull my end up 8.64 ft.

Too good to be true? YES! We forgot that friction comes along with every machine. There are no ideal machines. Friction is like pac man. It eats Mechanical (meaning useful) Energy, chews it up and excretes random heat.

Pac Man excretes random heat after chewing up the more organized mechanical energy which increases the disorder (chaos) of the surroundings. More about this later.



The red dots that Pac Man eats represent the Mechanical Energy (E) stored in a system.

If we want to get a little more real, we have to include pac man. Two ways to do this:

#1) Empirical (experimental) way: Determine Actual Mechanical Advantage (AMA) by actually applying a force at a point on a lever that moves a known amount of weight (load). Then just divide the Load (force lout) by your effort.

$$AMA = \frac{Force\ Out}{Force\ In} = \frac{Load}{Effort}$$

#2) Use the **efficiency** of the machine to determine the AMA from the IMA. Efficiency is a measure of how ideal or perfect a machine is. If the machine is perfect then Work out = Work in. Since $Efficiency = \frac{Work\ out}{Work\ in} = 1.$ we can expand and manipulate the equation:

$$Efficiency = \frac{F_{out}d_{out}}{F_{in}d_{in}} = \frac{F_{out}/F_{in}}{d_{in}/d_{out}} = \frac{AMA}{IMA}\ \ or\ \ Efficiency(IMA) = (AMA)$$

Obviously, the efficiency must be between 0 and 1. If there is very little friction, the efficiency is closer to one and your machine is close to ideal. If the efficiency is low, the machine has a lot of internal friction. The more complex the machine, the lower the efficiency. A good fuel efficient car has a percent efficiency around 15%. You can think of the car engine as a series of simple machines. A good motor oil can increase the percent efficiency of a motor by a few points. We'll get to compound machines in a bit.

One other efficiency equation that might come in handy is the efficiency of a motor: $Efficiency = \frac{actual\ power}{ideal\ power}$